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LEVELING

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Introduction

“Leveling” is a general term used in land surveying that applies to vertical measurements. Vertical measurements are made and referenced to datums, as elevations. The reference datum might be an arbitrary elevation chosen for convenience or a very precise value determined after lengthy studies. The standard reference datum used throughout California is mean sea level, based on the National Geodetic Vertical Datum (NGVD 1929).

Three methods used to measure differences in elevation are direct vertical measurement, trigonometric leveling, and differential leveling. It is important to understand the procedure, equipment and note keeping format used for each method.

Performance Expected on the Exams

Define the terms “curvature” and “refraction,” be able to calculate their combined effects and explain the procedure used to limit their effects.

Explain how to peg a level and calculate the collimation correction of a properly adjusted level.

Given field measurements, calculate the difference in elevation between two stations using trigonometric leveling method.

Explain, interpret, reduce and adjust differential leveling notes.

Explain, interpret, reduce and adjust three-wire leveling notes.

Obtain the difference in elevation between two stations by reciprocal leveling.

Plan and analyze the results of a leveling project with regard to NGS standards and specifications.

Key Terms

Altimetry	Lenker rod
Automatic pendulum level	Mean sea level
Backsight (+shot)	National Geodetic Vertical datum (NGVD 1929)
Bench mark	North American Vertical Datum (NAVD 1988)
Curvature	Pegging a level
Datum	Philadelphia rod
Direct leveling	Plumb line
Differential leveling	Profile leveling
Elevation	Reciprocal leveling
Foresight (-shot)	Refraction
H.I.	Three-wire leveling
Horizontal line	Trigonometric leveling
Horizontal plane	Turning point (TP)
Intermediate foresight (side shot)	Vertical difference
Leveling	Vertical line
Level surface	
Level line	

Video Presentation Outline

Basic Concepts

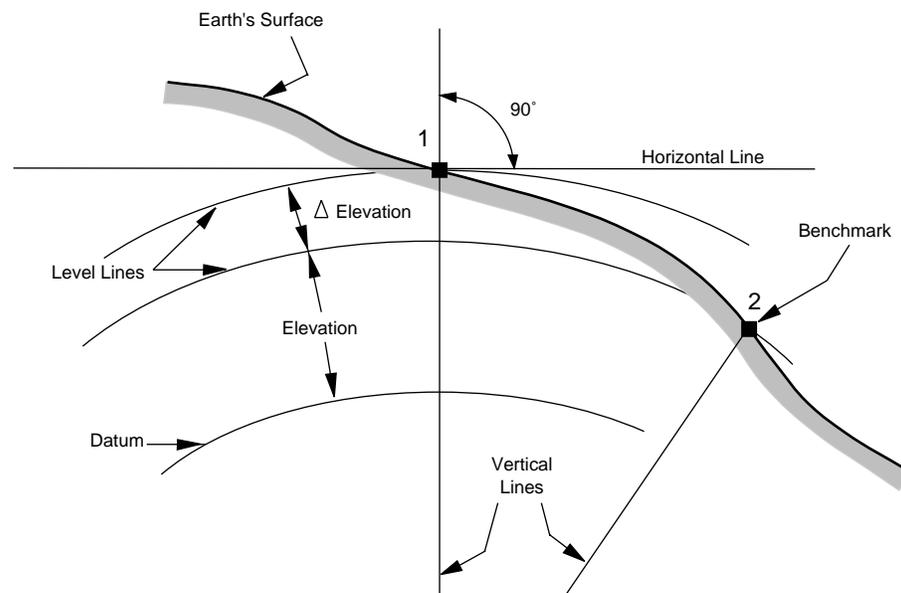


Figure 6-1. Leveling concepts.

- Level line
- Horizontal line
- Vertical line
- Datum

Curvature and Refraction

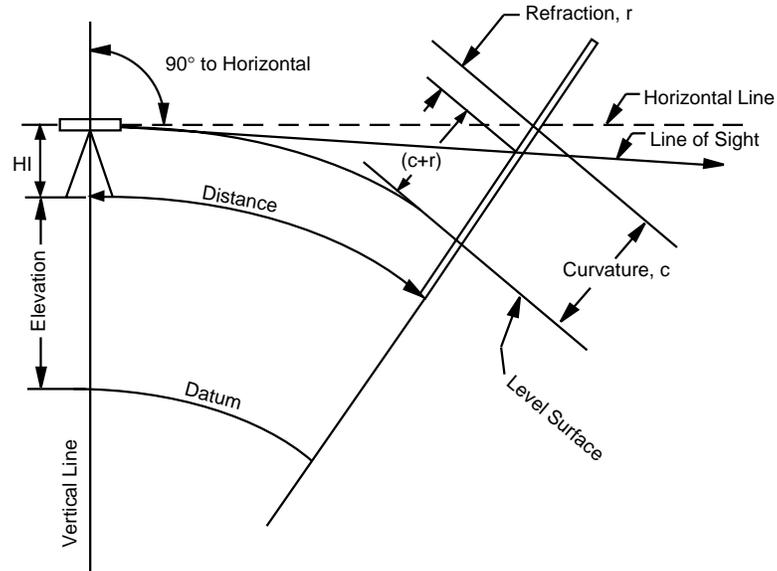


Figure 6-2. Curvature and refraction.

- Curvature
- Refraction
- The formula for computing the combined effect of curvature and refraction is:

$$C + R = 0.021K^2$$

C = correction for curvature

R = correction for refraction

K = sighting distance in thousands of feet

- Corrections for various distances

Distance	Correction
100'	0.00021'
200'	0.00082'
500'	0.0052'
700'	0.01'
1 mile	0.574'

Three Methods of Vertical Measurement Leveling

Direct Vertical Measurement Leveling

- Altimetry
- Direct elevation rods
- Lasers
- G.P.S.

Trigonometric Leveling

- Equipment
- Method
- Calculation

$$\text{Elev. "B"} = \text{Elev. "A"} + \text{H.I.} + (\cos Z) (S) - \text{rod} - (c+r)$$

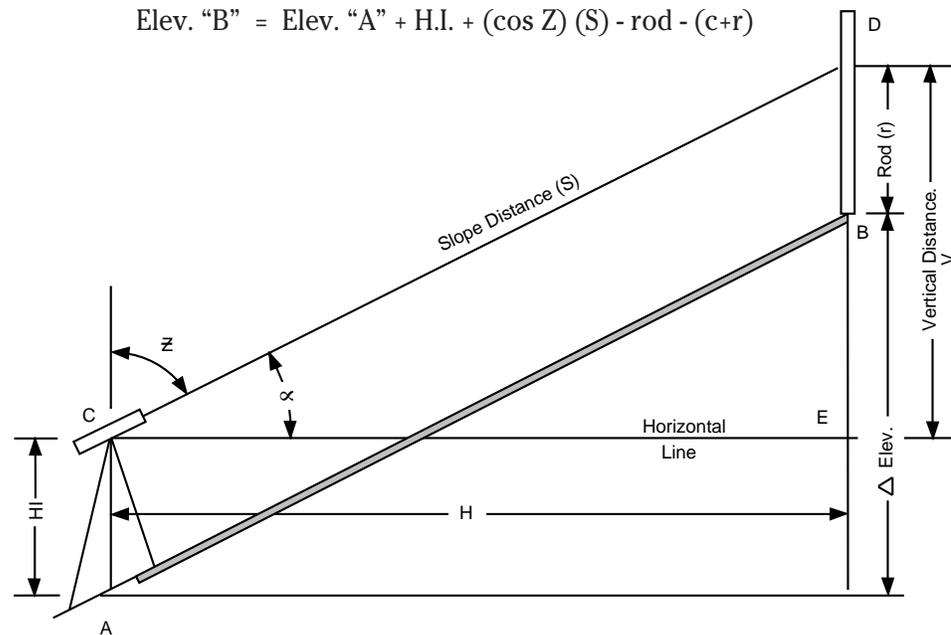


Figure 6-3. Trigonometric leveling.

Differential Leveling

- Equipment
- Method
- Calculations

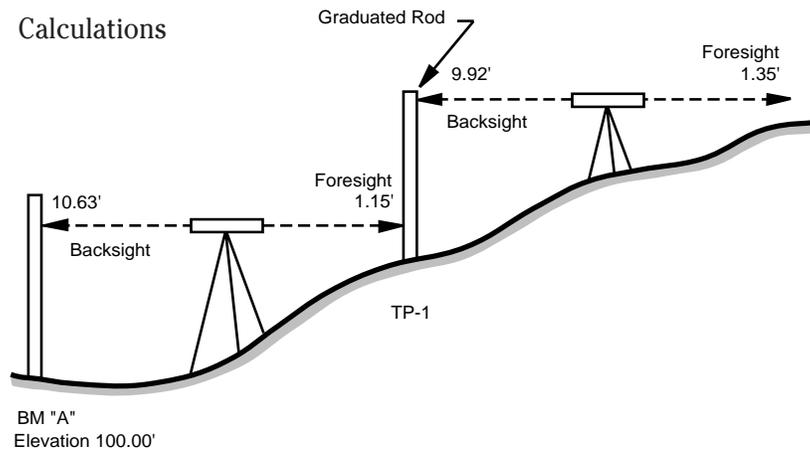


Figure 6-4. Differential leveling.

Notekeeping for Differential Leveling

- Standard notekeeping

	+	∩	-	Elev.
				(412.011)
B.M. #2			5.090	412.005
	7.330	417.095		
T.P.			4.765	409.765
	6.995	414.530		
B.M. #1				407.535

Figure 6-5. Profile leveling noteform.

Meter Rod

Precise (3-wire) Leveling (Metric Rods)		Leveling (Metric Rods)			
Set-up	Thread I.S.	Mean I.S.	1/2 I.S.	2 I.S.	Back-sight Interf. (in ft.)
1	2.557				
	2.565	2.8650	172		3.96
	2.198		172		
			344	344	
2	0.886				
	0.488	0.6880	171		2.24
	0.514	0.6880	171		10.00
			342	342	
3	0.850				
	0.789	0.7890	111		2.48
	0.628	0.7890	111		12.48
			222	222	
4	1.820				
	1.182	1.1820	128		3.92
	1.664	0.9810	128		16.28
			256	1168	
5	0.510				
	0.482	0.4820	060		1.42
	0.378	0.4820	067		17.80
			132	1896	
6	0.928				
	0.874	0.8740	058		2.87
	0.818	0.8740	057		+20.67
			115	1411	

Forward from BM 11 to BM 12					
Thread I.S.	Mean I.S.	1/2 I.S.	2 I.S.	Back-sight Interf. (in ft.)	Notes
1.788					Temp 34°C
1.852	1.8218	103		8.08	Family B
1.867		103			Edison A
		206	206		J. Powell B
2.708					C. Quirk B
2.858	2.8297	149		8.38	Wickoff B
2.385	0.1060	170		13.40	Level 140
		339	707		Metric Rod
2.622					Set 14
2.511	2.5110	111		8.24	
2.900	0.4176	111		21.70	
		222	829		
2.788					
2.618	2.6180	121		8.58	
2.098	0.2329	120		30.28	
		241	1170		
2.855					
2.778	2.7780	082		8.12	
2.623	0.0080	080		39.40	
		162	1886		
2.060					
2.868	2.8680	092		7.77	
2.277	0.8778	091		-8.17	
		182	1818	+20.67	
		-8.005		-26.809	

Precise (3-wire) Leveling (Metric Rods)		Leveling (Metric Rods)			
Set-up	Thread I.S.	Mean I.S.	1/2 I.S.	2 I.S.	Back-sight Interf. (in ft.)
7	2.545				
	2.452	2.4860	091		8.08
	2.368		091		
			182	182	
8	2.489				
	2.857	2.8310	092		7.64
	2.805	0.7810	092		15.71
			184	366	
9	2.897				
	2.705	2.7047	142		8.87
	2.562	2.4987	143		24.58
			285	651	
10	3.008				
	2.846	2.8460	168		8.84
	2.486	0.8417	168		33.92
			336	876	
11	2.880				
	2.080	2.0860	100		6.84
	1.980	0.2827	100		40.78
			200	1254	
12	1.065				
	1.028	1.0280	060		3.84
	0.888	0.4867	060		+06.18
			120	1380	
					-17.63
BM 12 to BM 11	0.0000				+20.67
BM 11 to BM 12	0.0000				-20.67

Return from BM 12 to BM 11					
Thread I.S.	Mean I.S.	1/2 I.S.	2 I.S.	Back-sight Interf. (in ft.)	Notes
0.585					
0.565	0.5647	080		1.65	
0.484		081			
		161	161		
0.539					
0.452	0.4520	087		1.48	
0.365	0.3647	087		8.13	
		174	335		
0.612					
0.452	0.4518	140		1.58	
0.350	1.4780	142		4.47	
		282	617		
0.870					
0.865	0.8647	165		1.38	
0.245	1.8347	166		6.01	
		330	942		
1.676					
1.808	1.8087	172		8.84	
1.831	3.3389	173		14.95	
		345	1287		
2.128					
2.039	2.0390	089		6.68	
1.950	0.5379	089		-17.63	
		178	1465		
Discrepancy:	2.6mm			(0.012)	
Mean O.K. Error:	0.0080			(0.002)	

Figure 6-7. Level notes for meter rod from noteforms.
(Reproduced with permission from Landmark Enterprises.)

Special Leveling Procedures

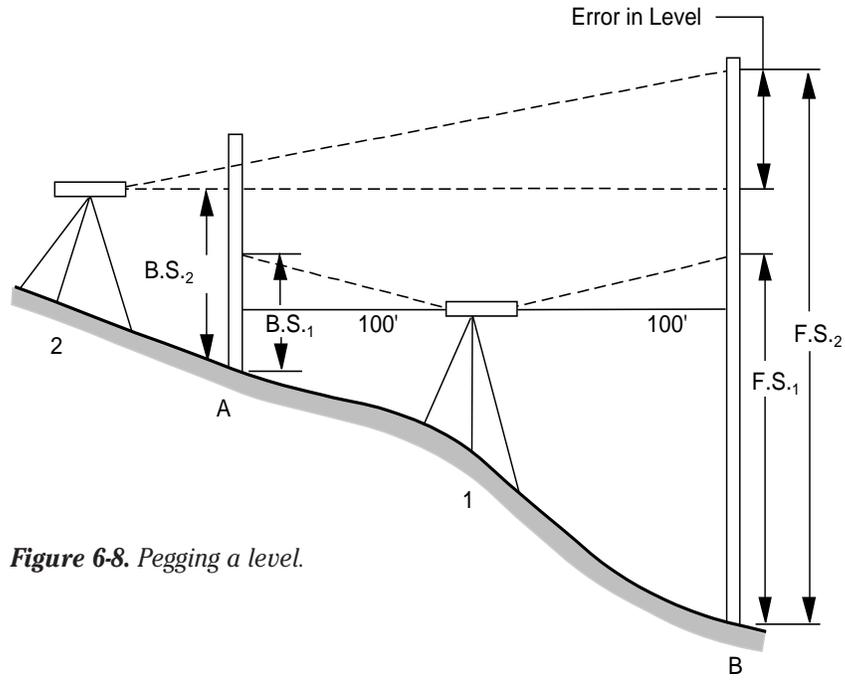


Figure 6-8. Pegging a level.

Sample Peg Test

Station	Backsight (+)	H.I.	Foresight	Elevation
A	5.10	105.10		100.00'
(assumed)				
B			4.96	100.14'
A	5.51	105.51		100.00'
B			5.35	100.16'

Adjustment = elevation of B from setup 1 - elevation of B from setup 2.

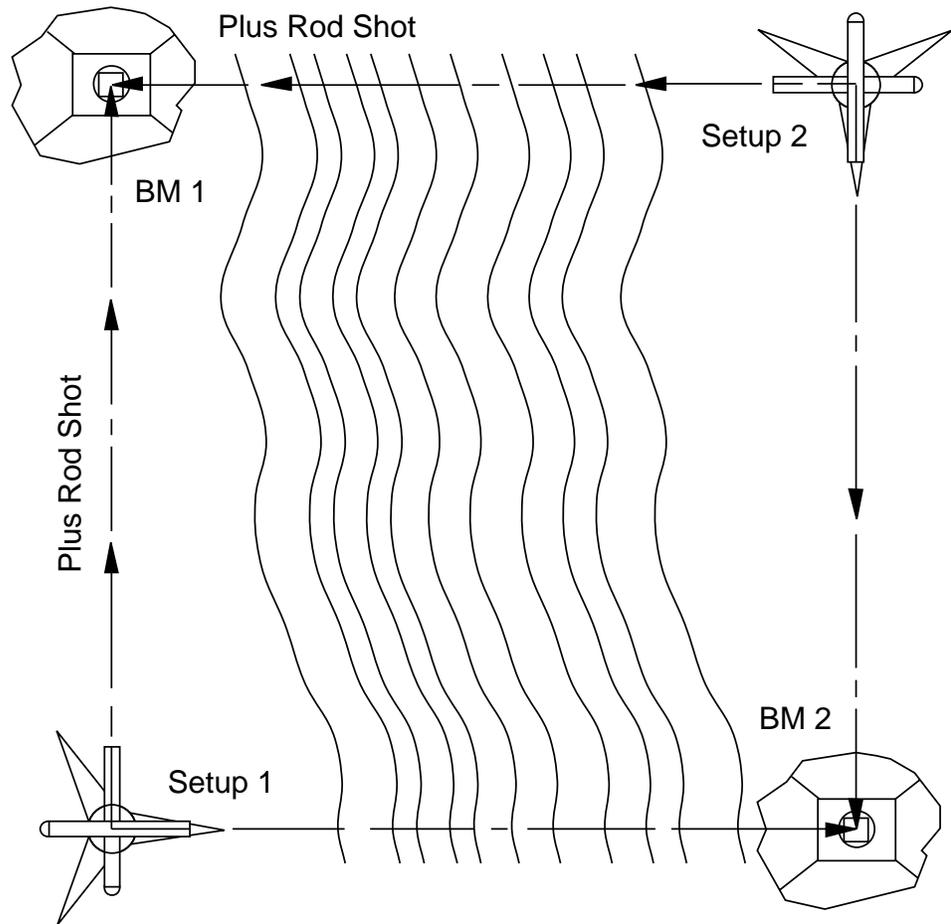


Figure 6-9. Reciprocal leveling.

Classification of Accuracy Standards and Adjustments

General Specifications for Vertical Control Field Procedures

Order Class	First I	First II	Second I	Second II	Third
Minimal Observation Method	Micrometer	Micrometer	Micrometer or Three-Wire	Three-Wire	Center Wire
Section Running	DR, DS, or MDS	DR, DS, or MDS	DR	DR	DR
Difference of forward and backward sight lengths never to exceed:					
per setup (m)	2	5	5	10	10
per section (m)	4	10	10	10	10
Maximum sight length (m)	50	60	60	70	90
Minimum ground clearance of line of sight (m)	0.5	0.5	0.5	0.5	0.5
Even no. of setups when not using leveling rods with detailed calibration	yes	yes	yes	yes	—
Determine temp. gradient for vert. range of line of sight for each setup	yes	yes	yes	—	—
Maximum section misclosure (mm)	$3\sqrt{D}$	$4\sqrt{D}$	$6\sqrt{D}$	$8\sqrt{D}$	$12\sqrt{D}$
Maximum loop misclosure (mm)	$4\sqrt{E}$	$5\sqrt{E}$	$6\sqrt{E}$	$8\sqrt{E}$	$12\sqrt{E}$
Single-Run Methods					
Reverse direction of single runs every half day	yes	yes	yes	—	—
Non-reversible compensator leveling instruments					
Off-level/relevel instrument bet. observing the high and low rod scales	yes	yes	yes	—	—
Three-Wire Method					
Reading check (difference between top and bottom intervals) for one setup not to exceed (tenths of rod units)	—	—	2	2	3
Read rod first in alternate setup method	—	—	yes	yes	yes
Double Scale Rods					
Low-high scale elevation difference for one setup not to exceed (mm)					
With reversible compensator	0.40	1.00	1.00	2.00	2.00
Other instrument types:					
Half-centimeter rods	0.25	0.30	0.60	0.70	1.30
Full-centimeter rods	0.30	0.30	0.60	0.70	1.30
DS-Double Simultaneous procedure					
DR-Double-Run					
MDS-Modified Double Simultaneous					
D-shortest length of section (one-way) in km.					
E-perimeter of loop in km.					
# Must double-run when using three-wire method					
* May single-run if line length between network control points is less than 25 km and may single-run if line length between network control points is less than 10 km.					

NOTE: See *Geometric Geodetic Accuracy Standards and Specifications for Using GPS Relative Positioning Techniques*, Federal Geodetic Control Committee for latest specifications.

Figure 6-10. "General Specifications for Vertical Control," National Geodetic Survey.

Adjustments to Level Runs

- Length of lines methods
- Number of turning points method
- Least squares method

Sample Test Questions

1. When pegging a level the surveyor reads 5.25 on the backsight rod and 5.38 on the foresight rod. After moving the level adjacent to the backsight rod, a reading of 5.18 is taken on the near rod. What should be on the far rod?
2. When pegging a level, how far apart should the rod readings be taken?
3. The effects eliminated by keeping backsights and foresights equal are _____ and _____.
4. Does an instrument in perfect adjustment sight a level line to a distant object? Explain.
5. Fill in the missing data in the sample differential level run below.

Station	B.S. (+)	H.I.	F.S. (-)	Elevation
B.M. A				256.18
	4.05	<u>a.</u>		
T.P. 1			6.48	253.75
			10.26	<u>b.</u> (top Pipe)
	5.26	259.01		
T.P. 2			7.56	<u>c.</u>
	2.56	254.01		
T.P. 3			<u>d.</u>	245.56
	<u>e.</u>	250.45		
B.M. B			7.08	<u>f.</u> (243.39)

-
6. What is the misclosure in the sample differential level run in problem 5? What is the adjusted elevation of T.P. 2?
 7. To meet Class II, Second Order accuracies, what would be the maximum misclosure of a level run of two kilometers?
 8. What is the recommended leveling method for meeting the Class II Second Order Standard?
 9. Is it necessary to balance the foresights and backsights to achieve the necessary accuracies in question 7? If yes, what is the maximum difference allowed per setup? What is the maximum length allowed per sight?
 10. A theodolite is set up over Point 123 with an H.I. of 5.59 feet. The elevation of Point 123 is 2105.67 feet. The measured zenith angle to a target with an H.I. of 4.77 at Point 124, is $94^{\circ} 35' 46''$. The slope distance measured from the theodolite to the target is 2145.89 feet. What is the difference in elevation between Point 123 and Point 124? What field procedure could you use that would allow you to discount the effects of curvature and refraction on the results?
 11. Problem D-6, 1978 LS
Problem Statement: A collection of rod readings is shown below. These readings were taken over a section of line of three-wire levels run in both directions using a precision self-leveling level and invar-faced-rods graduated in centimeters with readings estimated to the nearest millimetre. The C-factor of the instrument is -0.150 , the stadia constant is 0.335 and the average rod temperature is 30°C .

Required:
 - A. Arrange these notes in the workbook paper as would be done in a field book, showing all data normally shown in field notes for precise leveling.
 - B. Reduce and analyze the notes, showing all intermediate steps and checks. Note any deviations from acceptable practice and/or limits, and proceed to a determination of the mean difference in elevation for the section and assumptions you make. Compute and apply all applicable corrections for systematic errors. Express the difference in elevation in meters.
 - C. Determine and state the highest order of leveling for which this run would qualify. Use the latest published standards for vertical control surveys.
-

D. Discuss the concept of orthometric corrections: What it is, what it does, when it is used, and the kind of work to which it is normally applied. Would it be likely to be applied to the data in this problem? Why?

Rod Readings		Three Wire Levels	
Forward Run		Backward Run	
+	-	+	-
234	2392	1831	721
198	2359	1802	693
162	2327	1775	666
1455	1629	3077	171
1384	1557	3057	151
1313	1484	3037	131
158	3250	3332	298
155	3227	3310	227
112	3203	3288	257
126	3125	1854	1811
117	3106	1784	1742
088	3087	1714	1672
808	1832	2603	240
777	1795	2563	202
747	1779	2542	164

Answer Key

1. 5.31 ft
2. 200 ft
3. Curvature, refraction
4. No, the line of sight is a curved line due to atmospheric refraction.
5.
 - A. 260.23
 - B. 249.97
 - C. 251.45
 - D. 8.45
 - E. 4.89
 - F. 243.37
6. Misclosure = -.02 ft., t.p.#2 = 251.46
7. 11.31 mm
8. Three-wire, double-run or single-run double simultaneous procedure
9. Yes, 10 m, maximum sight length is 70 m
10. Change in
Elevation = H.I. PT. 123 - H.I. PT 124 + ((cos Z) (slope dist.) - (c+r))
= 5.59 - 4.77 + (cos 94° 35' 46" x 2145.89) - (.021 x 2.14589²)
= -171.23 ft

The effect of curvature and refraction will be canceled if measurements are made from each end of the line, and the mean of the results of the two sets of measurements is used.

11. A. Refer to Figure 6-7 for field noteform for three-wire level notes using a meter rod.
- B. Assume $K = 100$

$$\begin{aligned} \text{Forward run} \\ \text{distance leveled} &= \text{B.S. Intervals} + \text{F.S. Intervals} + 10 \\ &\quad (\text{Stadia Constant}) \\ &= 359 + 367 + 10 (.335) \\ &= 729.4\text{M} \\ &= .7294 \text{ km} \end{aligned}$$

$$\begin{aligned} \text{Corrected elev.} &= \text{F.S.} + \text{B.S.} \\ &= 2.6013 + (-12.0440) \\ &= -9.4427 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Correction for} \\ \text{interval imbalance} &= \text{Difference between F.S. and} \\ &\quad \text{B.S. Intervals} \times -0.15 \\ &= 8 \times -0.15 \text{ mm} \\ &= -1.2 \text{ mm} \end{aligned}$$

$$\text{Elev.} = -9.4427 - .0012 = -9.4439$$

$$\begin{aligned} \text{Backward run} \\ \text{distance leveled} &= 712 \text{ M} + 10 (.335) = .7154 \text{ km} \end{aligned}$$

$$\begin{aligned} \text{Corrected elev.} &= \frac{9.4510 - 10 \times .15}{1000} \\ &= 9.4495 \text{ m} \end{aligned}$$

There are two bad rod readings in the notes, both in the forward run.

1. The foresight middle wire reading at STA. 4 should be 107 rather than 117.
2. The backsight low wire reading at STA. 5 should be 1759 rather than 1779.

C. Average length
 of section = Mean of forward and backward runs
 = .7224 km

Average difference
in elevation = 9.4467

Difference in
elevation between
forward and
backward runs = 9.4439 - 9.4495
 = .0056 m
 = 5.6 mm

Difference in forward and backward sight lengths < 10 m and maximum section misclosures from Figure 6-10 show this run meets Second Order, Class II requirements.

- D. Reference: *The Surveying Handbook*, Brinker and Minnick and *Caltrans Surveys Manual*.

Level surfaces are perpendicular to the direction of gravity. Gravity is affected by the variation of centrifugal force which increases with altitude and decreasing latitude. In geodetic leveling, this variation in gravity accounts for nonparallel level surfaces. Orthometric correction is applied to account for convergence of level surfaces for long level runs in north-south directions or runs at high elevations.

Orthometric correction would not be applied to the data in this problem because required information for application of orthometric correction such as latitude and elevation are not given, and short level runs of this order and class would be unaffected by orthometric correction.

References

- _____, *Surveys Manual*, California Department of Transportation, Chapters 2, 4, and 5.
- _____, *Definitions of Surveying and Associated Terms*, A.C.S.M., Bethesda, Maryland, 1978.
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- Wolf, Paul R., and Brinker, Russell C., *Elementary Surveying*, Eighth Edition, Harper & Row, New York, 1989. (A very good presentation of the subject.)